

Time-dependent Dalitz-plot formalism for $B_q^0 \rightarrow J/\psi h^+ h^-$

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Abstract

A formalism for measuring time-dependent CP violation in $B_{(s)}^0 \rightarrow J/\psi h^+ h^-$ decays with $J/\psi \rightarrow \mu^+ \mu^-$ is developed for the general case where there can be many $h^+ h^-$ final states of different angular momentum present. Here h refers to any spinless meson. The decay amplitude is derived using similar considerations as those in a Dalitz like analysis of three-body spinless mesons taking into account the fact that the J/ψ is spin-1, and the various interferences allowed between different final states. Implementation of this procedure can, in principle, lead to the use of a larger number of final states for CP violation studies.

1 Introduction

Measurement of CP violation in the B^0 and B_s^0 systems is important for testing the Standard Model, as new particles can appear in mixing diagrams. Previous measurements have been made in many modes [1]. To measure the phase in B_s^0 decays the final states $B_s^0 \rightarrow J/\psi K^+ K^-$ for $K^+ K^-$ masses close to that of the ϕ meson has been used [2–4], as well as $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$ [5]. In the latter case the final state is CP odd [6] over most of the $\pi^+ \pi^-$ mass range, while in the case of $K^+ K^-$ the final state even in the mass region near the ϕ meson has both CP odd and even components, that can be resolved using time-dependent angular analysis [7]. In this paper we present a formalism that allows the entire $K^+ K^-$ mass region to be used in CP violation measurements regardless of the final state angular momentum. This formalism can also be applied to B^0 decays, e.g. $B^0 \rightarrow J/\psi \pi^+ \pi^-$.

The basic concept here is to couple a three-body Dalitz like analysis [8] to the $J/\psi h^+ h^-$ final state, where the $J/\psi \rightarrow \mu^+ \mu^-$ and concurrently measure the time-dependent CP violation by splitting the final state into odd and even CP components.

2 Time-dependent decay rates

The time evolution of the $B_q^0 - \bar{B}_q^0$ system is described by the Schrödinger equation

$$i \frac{\partial}{\partial t} \begin{pmatrix} |B_q^0(t)\rangle \\ |\bar{B}_q^0(t)\rangle \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2} \mathbf{\Gamma} \right) \begin{pmatrix} |B_q^0(t)\rangle \\ |\bar{B}_q^0(t)\rangle \end{pmatrix}, \quad (1)$$

where the \mathbf{M} and $\mathbf{\Gamma}$ matrices are Hermitian, and CPT invariance implies that $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. The off-diagonal elements, M_{12} and Γ_{12} , of these matrices describe the off-shell (dispersive) and on-shell (absorptive) contributions to $B_q^0 - \bar{B}_q^0$ mixing, respectively.

The mass eigenstates $|B_H\rangle$ and $|B_L\rangle$ of the effective Hamiltonian matrix are given by

$$\begin{aligned} |B_L\rangle &= p|B_q^0\rangle + q|\bar{B}_q^0\rangle, \\ |B_H\rangle &= p|B_q^0\rangle - q|\bar{B}_q^0\rangle, \end{aligned} \quad (2)$$

with $|p|^2 + |q|^2 = 1$. The decay amplitudes for B_q^0 and \bar{B}_q^0 into a self-charge-conjugated final state f , where for this paper $f = J/\psi h^+ h^-$, are defined as

$$A_f \equiv \langle f | S | B_q^0 \rangle, \quad \bar{A}_f \equiv \langle f | S | \bar{B}_q^0 \rangle. \quad (3)$$

With the additional definitions

$$A \equiv A_f, \quad \text{and} \quad \bar{A} \equiv \frac{q}{p} \bar{A}_f, \quad (4)$$

the time dependent decay rates can be written as [9]

$$\Gamma(t) = \mathcal{N} e^{-\Gamma t} \left\{ \frac{|A|^2 + |\bar{A}|^2}{2} \cosh \frac{\Delta\Gamma t}{2} + \frac{|A|^2 - |\bar{A}|^2}{2} \cos(\Delta m t) - \mathcal{R}e(A^* \bar{A}) \sinh \frac{\Delta\Gamma t}{2} - \mathcal{I}m(A^* \bar{A}) \sin(\Delta m t) \right\}, \quad (5)$$

$$\bar{\Gamma}(t) = \left| \frac{p}{q} \right|^2 \mathcal{N} e^{-\Gamma t} \left\{ \frac{|A|^2 + |\bar{A}|^2}{2} \cosh \frac{\Delta\Gamma t}{2} - \frac{|A|^2 - |\bar{A}|^2}{2} \cos(\Delta m t) - \mathcal{R}e(A^* \bar{A}) \sinh \frac{\Delta\Gamma t}{2} + \mathcal{I}m(A^* \bar{A}) \sin(\Delta m t) \right\}, \quad (6)$$

where \mathcal{N} is a normalization constant, $\Delta m = m_H - m_L$, $\Delta\Gamma = \Gamma_L - \Gamma_H$, and $\Gamma = (\Gamma_L + \Gamma_H)/2$.

3 Angular dependent formulas

3.1 Definition of helicity angles

We express the angular dependence of the decay in terms of “helicity” angles defined as (i) θ_ℓ , the angle between the μ^+ direction in the J/ψ rest frame with respect to the J/ψ direction in the B_q^0 rest frame; (ii) θ_h the angle between the h^+ direction in the h^+h^- rest frame with respect to the h^+h^- direction in the B_q^0 rest frame, and (iii) χ the angle between the J/ψ and h^+h^- decay planes in the B_q^0 rest frame. These angles are shown pictorially in Fig. 1. (These definitions are the same for B_q^0 and \bar{B}_q^0 , namely, using μ^+ and h^+ to define the angles for both B_q^0 and \bar{B}_q^0 decays.)

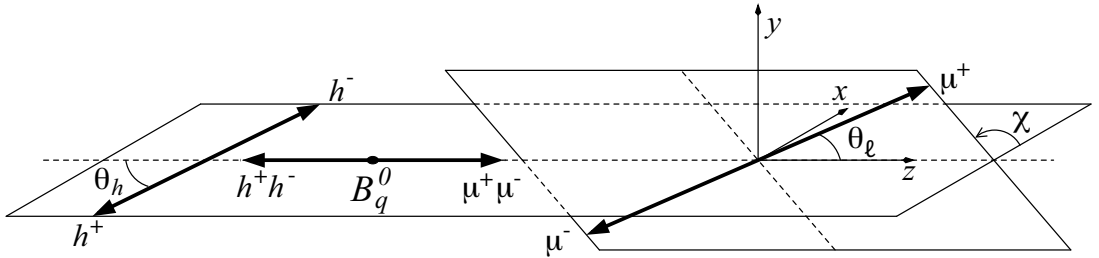


Figure 1: Definition of helicity angles. For details see text.

3.2 Time-independent part of the rate for B_q^0 decays

For the decays of $B_q^0 \rightarrow J/\psi h^+ h^-$ with $J/\psi \rightarrow \mu^+ \mu^-$ the decay rate is found by summing over the unobserved lepton polarizations. The time-independent part of the rate is¹

$$|A_f(m_{hh}, \theta_h, \theta_\ell, \chi)|^2 = \sum_{\alpha=\pm 1} \left| \sum_{\lambda, J}^{| \lambda | \leq J} \sqrt{\frac{2J+1}{4\pi}} H_\lambda^J(m_{hh}) e^{i\lambda\chi} d_{\lambda, \alpha}^1(\theta_\ell) d_{-\lambda, 0}^J(\theta_h) \right|^2, \quad (7)$$

where $\lambda = 0, \pm 1$ is the J/ψ helicity, $\alpha = \pm 1$ is the helicity difference between the two muons, J is the spin of the $h^+ h^-$ intermediate state, and $H_\lambda^J(m_{hh})$ is a helicity amplitude depending on m_{hh} that can be expressed using a formalism similar to that in a Dalitz-plot analyses. We define the term which contains the sum over spin- J as

$$\mathcal{H}_\lambda(m_{hh}, \theta_h) = \sum_J \sqrt{\frac{2J+1}{4\pi}} H_\lambda^J(m_{hh}) d_{-\lambda, 0}^J(\theta_h). \quad (8)$$

Then Eq. (7) becomes

$$\begin{aligned} |A_f(m_{hh}, \theta_h, \theta_\ell, \chi)|^2 &= \sum_{\alpha=\pm 1} \left| \sum_{\lambda} e^{i\lambda\chi} d_{\lambda, \alpha}^1(\theta_\ell) \mathcal{H}_\lambda(m_{hh}, \theta_h) \right|^2 \\ &= \sum_{\alpha=\pm 1} \left[\left(\sum_{\lambda'} e^{i\lambda'\chi} d_{\lambda', \alpha}^1(\theta_\ell) \mathcal{H}_{\lambda'}(m_{hh}, \theta_h) \right)^* \left(\sum_{\lambda} e^{i\lambda\chi} d_{\lambda, \alpha}^1(\theta_\ell) \mathcal{H}_\lambda(m_{hh}, \theta_h) \right) \right] \\ &= \sum_{\lambda', \lambda} \left(\sum_{\alpha=\pm 1} d_{\lambda', \alpha}^1(\theta_\ell) d_{\lambda, \alpha}^1(\theta_\ell) \right) \mathcal{H}_{\lambda'}^*(m_{hh}, \theta_h) \mathcal{H}_\lambda(m_{hh}, \theta_h) e^{i(\lambda-\lambda')\chi}. \end{aligned} \quad (9)$$

Defining

$$\Theta_{\lambda'\lambda}(\theta_\ell) \equiv \sum_{\alpha=\pm 1} d_{\lambda', \alpha}^1(\theta_\ell) d_{\lambda, \alpha}^1(\theta_\ell), \quad (10)$$

results in

$$|A_f(m_{hh}, \theta_h, \theta_\ell, \chi)|^2 = \sum_{\lambda', \lambda} \mathcal{H}_\lambda(m_{hh}, \theta_h) \mathcal{H}_{\lambda'}^*(m_{hh}, \theta_h) e^{i(\lambda-\lambda')\chi} \Theta_{\lambda'\lambda}(\theta_\ell). \quad (11)$$

Table 1 lists the functions $\Theta_{\lambda'\lambda}(\theta_\ell)$. They are invariant under the interchange of λ and λ' , i.e. $\Theta_{\lambda'\lambda}(\theta_\ell) = \Theta_{\lambda\lambda'}(\theta_\ell)$, and transform with respect to a change of the sign of both λ and λ' as $\Theta_{\lambda\lambda'}(\theta_\ell) = (-1)^{\lambda-\lambda'} \Theta_{-\lambda'-\lambda}(\theta_\ell)$. Inserting the explicit functional forms in Eq. (11) allows us to express the amplitude as

$$\begin{aligned} |A_f(m_{hh}, \theta_h, \theta_\ell, \chi)|^2 &= |\mathcal{H}_0(m_{hh}, \theta_h)|^2 \sin^2 \theta_\ell + \frac{1}{2} (|\mathcal{H}_+(m_{hh}, \theta_h)|^2 + |\mathcal{H}_-(m_{hh}, \theta_h)|^2) \\ &\quad \times (1 + \cos^2 \theta_\ell) + \mathcal{R}e [\mathcal{H}_+(m_{hh}, \theta_h) \mathcal{H}_-^*(m_{hh}, \theta_h) e^{2i\chi}] \sin^2 \theta_\ell \\ &\quad + \sqrt{2} \mathcal{R}e [(\mathcal{H}_0(m_{hh}, \theta_h) \mathcal{H}_+^*(m_{hh}, \theta_h) - \mathcal{H}_0^*(m_{hh}, \theta_h) \mathcal{H}_-(m_{hh}, \theta_h)) e^{-i\chi}] \\ &\quad \times \sin \theta_\ell \cos \theta_\ell, \end{aligned} \quad (12)$$

¹In $d_{-\lambda, 0}^J(\theta_h)$, $-\lambda$ is used instead of λ in order to be consistent with the convention used in [3].

where we denote \mathcal{H}_λ by 0, +, and −, rather than 0, +1 and −1.

Table 1: Functional forms of $\Theta_{\lambda'\lambda}(\theta)$ defined in Eq. (10) for different values of λ and λ' .

λ	λ'	$\Theta_{\lambda'\lambda}(\theta)$
0	0	$\sin^2 \theta$
0	1	$\frac{1}{\sqrt{2}} \sin \theta \cos \theta$
0	−1	$-\frac{1}{\sqrt{2}} \sin \theta \cos \theta$
1	0	$\frac{1}{\sqrt{2}} \sin \theta \cos \theta$
1	1	$\frac{1}{2}(1 + \cos^2 \theta)$
1	−1	$\frac{1}{2} \sin^2 \theta$
−1	0	$-\frac{1}{\sqrt{2}} \sin \theta \cos \theta$
−1	1	$\frac{1}{2} \sin^2 \theta$
−1	−1	$\frac{1}{2}(1 + \cos^2 \theta)$

3.3 Time-independent part of the rate for \overline{B}_q^0 decays

For \overline{B}_q^0 decays, the expression for $|\overline{A}_f(m_{hh}, \theta_h, \theta_\ell, \chi)|^2$, results from replacing $\mathcal{H}_\lambda(m_{hh}, \theta_h)$ in Eq. (12) by $\overline{\mathcal{H}}_\lambda(m_{hh}, \theta_h)$, which contains the helicity amplitudes for \overline{B}_q^0 decays. $\mathcal{H}_\lambda(m_{hh}, \theta_h)$ and $\overline{\mathcal{H}}_\lambda(m_{hh}, \theta_h)$ are related by transversity CP eigenstates [10], that are discussed in Section 4. Using these we find

$$\begin{aligned}
|\overline{A}_f(m_{hh}, \theta_h, \theta_\ell, \chi)|^2 &= |\overline{\mathcal{H}}_0(m_{hh}, \theta_h)|^2 \sin^2 \theta_\ell + \frac{1}{2} (|\overline{\mathcal{H}}_+(m_{hh}, \theta_h)|^2 + |\overline{\mathcal{H}}_-(m_{hh}, \theta_h)|^2) \\
&\quad \times (1 + \cos^2 \theta_\ell) + \mathcal{R}e \left[\overline{\mathcal{H}}_+(m_{hh}, \theta_h) \overline{\mathcal{H}}_-^*(m_{hh}, \theta_h) e^{2i\chi} \right] \sin^2 \theta_\ell \\
&\quad + \sqrt{2} \mathcal{R}e \left[\left(\overline{\mathcal{H}}_0(m_{hh}, \theta_h) \overline{\mathcal{H}}_+^*(m_{hh}, \theta_h) - \overline{\mathcal{H}}_0^*(m_{hh}, \theta_h) \overline{\mathcal{H}}_-(m_{hh}, \theta_h) \right) e^{-i\chi} \right] \\
&\quad \times \sin \theta_\ell \cos \theta_\ell .
\end{aligned} \tag{13}$$

3.4 The interference term

Next we calculate the complex term $A_f^*(m_{hh}, \theta_h, \theta_\ell, \chi) \overline{A}_f(m_{hh}, \theta_h, \theta_\ell, \chi)$. We have

$$\begin{aligned}
A_f^*(m_{hh}, \theta_h, \theta_\ell, \chi) \overline{A}_f(m_{hh}, \theta_h, \theta_\ell, \chi) &= \\
&\quad \sum_{\alpha=\pm 1} \left[\left(\sum_{\lambda'} e^{i\lambda'\chi} d_{\lambda',\alpha}^1(\theta_\ell) \mathcal{H}_{\lambda'}(m_{hh}, \theta_h) \right)^* \left(\sum_{\lambda} e^{i\lambda\chi} d_{\lambda,\alpha}^1(\theta_\ell) \overline{\mathcal{H}}_\lambda(m_{hh}, \theta_h) \right) \right] \\
&= \sum_{\lambda',\lambda} \overline{\mathcal{H}}_\lambda(m_{hh}, \theta_h) \mathcal{H}_{\lambda'}^*(m_{hh}, \theta_h) e^{i(\lambda-\lambda')\chi} \Theta_{\lambda'\lambda}(\theta_\ell).
\end{aligned} \tag{14}$$

Replacing the explicit terms leads to

$$\begin{aligned}
A_f^*(m_{hh}, \theta_h, \theta_\ell, \chi) \bar{A}_f(m_{hh}, \theta_h, \theta_\ell, \chi) &= \bar{\mathcal{H}}_0(m_{hh}, \theta_h) \mathcal{H}_0^*(m_{hh}, \theta_h) \sin^2 \theta_\ell \\
&+ \frac{1}{2} (\bar{\mathcal{H}}_+(m_{hh}, \theta_h) \mathcal{H}_+^*(m_{hh}, \theta_h) + \bar{\mathcal{H}}_-(m_{hh}, \theta_h) \mathcal{H}_-^*(m_{hh}, \theta_h)) (1 + \cos^2 \theta_\ell) \\
&+ \frac{1}{2} (\bar{\mathcal{H}}_+(m_{hh}, \theta_h) \mathcal{H}_-^*(m_{hh}, \theta_h) e^{2i\chi} + \bar{\mathcal{H}}_-(m_{hh}, \theta_h) \mathcal{H}_+^*(m_{hh}, \theta_h) e^{-2i\chi}) \sin^2 \theta_\ell \\
&+ \frac{1}{\sqrt{2}} (\bar{\mathcal{H}}_0(m_{hh}, \theta_h) \mathcal{H}_+^*(m_{hh}, \theta_h) e^{-i\chi} - \bar{\mathcal{H}}_0(m_{hh}, \theta_h) \mathcal{H}_-^*(m_{hh}, \theta_h) e^{i\chi} \\
&+ \bar{\mathcal{H}}_+(m_{hh}, \theta_h) \mathcal{H}_0^*(m_{hh}, \theta_h) e^{i\chi} - \bar{\mathcal{H}}_-(m_{hh}, \theta_h) \mathcal{H}_0^*(m_{hh}, \theta_h) e^{-i\chi}) \sin \theta_\ell \cos \theta_\ell. \quad (15)
\end{aligned}$$

4 Time-dependent Dalitz-plot formalism

Here we discuss the general formalism which includes S, P, D or higher waves of the h^+h^- intermediate states.

Apart from the proper decay-time t , the decay of $B_q^0 \rightarrow J/\psi h^+h^-$, $J/\psi \rightarrow \mu^+\mu^-$ can be described by four variables, we choose to use m_{hh} and three helicity angles $(\theta_\ell, \theta_h, \chi)$, where $(m_{hh}, \cos \theta_h)$ space is used instead of the usual variables in a Dalitz-plot analysis: $m_{hh}^2, m_{J/\psi h^+}^2$; the advantage is the former has an rectangle phase space which is easier for calculating the normalization.

Assuming $|p/q| = 1$, the differential decay rates in Eqs. (5) and (6) can be written in terms of the five variables $t, m_{hh}, \theta_\ell, \theta_h, \chi$ as

$$\begin{aligned}
\frac{d^5\Gamma}{dt dm_{hh} d\cos \theta_\ell d\cos \theta_h d\chi} &\propto e^{-\Gamma t} \left\{ \frac{|A_f|^2 + |\bar{A}_f|^2}{2} \cosh \frac{\Delta\Gamma t}{2} + \frac{|A_f|^2 - |\bar{A}_f|^2}{2} \cos(\Delta mt) \right. \\
&\quad \left. - \mathcal{R}e \left(\frac{q}{p} A_f^* \bar{A}_f \right) \sinh \frac{\Delta\Gamma t}{2} - \mathcal{I}m \left(\frac{q}{p} A_f^* \bar{A}_f \right) \sin(\Delta mt) \right\}, \quad (16)
\end{aligned}$$

$$\begin{aligned}
\frac{d^5\bar{\Gamma}}{dt dm_{hh} d\cos \theta_\ell d\cos \theta_h d\chi} &\propto e^{-\Gamma t} \left\{ \frac{|A_f|^2 + |\bar{A}_f|^2}{2} \cosh \frac{\Delta\Gamma t}{2} - \frac{|A_f|^2 - |\bar{A}_f|^2}{2} \cos(\Delta mt) \right. \\
&\quad \left. - \mathcal{R}e \left(\frac{q}{p} A_f^* \bar{A}_f \right) \sinh \frac{\Delta\Gamma t}{2} + \mathcal{I}m \left(\frac{q}{p} A_f^* \bar{A}_f \right) \sin(\Delta mt) \right\}. \quad (17)
\end{aligned}$$

The functions $|A_f|^2$, $|\bar{A}_f|^2$ and $A_f^* \bar{A}_f$ are defined in Eqs. (12), (13) and (15) respectively. We now substitute in Eq. (8) explicit variables for $H_\lambda^J(m_{hh})$ in terms of our chosen Dalitz-plot variables m_{hh} and θ_h , resulting in

$$\mathcal{H}_\lambda^{(-)}(m_{hh}, \theta_h) = \sum_R \mathbf{h}_\lambda^R \sqrt{2J_R + 1} \sqrt{P_R P_B} F_B^{(L_B)} F_R^{(L_R)} A_R(m_{hh}) \left(\frac{P_B}{m_B} \right)^{L_B} \left(\frac{P_R}{m_{hh}} \right)^{L_R} d_{-\lambda, 0}^{J_R}(\theta_h), \quad (18)$$

where the function $A_R(m_{hh})$ describes the mass squared shape of the resonance R , that in most cases is a Breit-Wigner function, P_B is the J/ψ momentum in the \bar{B}_q^0 rest frame, P_R is the momentum of either of the two hadrons in the dihadron rest frame, m_B is the

Table 2: CP parity for different spin resonances. Note that spin-0 only has one transversity component-0.

Spin	η_0	η_{\parallel}	η_{\perp}
0	-1	-	-
1	1	1	-1
2	-1	-1	1

\overline{B}_q^0 mass, L_B is the orbital angular momentum between the J/ψ and h^+h^- system, and L_R the orbital angular momentum in the h^+h^- decay, and thus is the same as the spin of the h^+h^- resonance. $F_B^{(L_B)}$ and $F_R^{(L_R)}$ are the Blatt-Weisskopf barrier factors for \overline{B}_q^0 and R resonance respectively [6].

The factor $\sqrt{P_R P_B}$ results from converting the phase space of the natural Dalitz-plot variables m_{hh}^2 and $m_{J/\psi h^+}^2$ to that of m_{hh} and $\cos\theta_h$. $\mathcal{H}_{\lambda}^{(-)}$ is summed over all h^+h^- intermediate states (R) with different spins, denoted as J_R . The function defined in Eq. (18) is based on previous Dalitz plot analyses [6, 11], but here all allowed values of L_B and L_R are included.

In order to use CP relations, it is convenient to replace the helicity complex coefficients $\mathbf{h}_{\lambda}^{(-)R}$ by the transversity complex coefficients $\mathbf{a}_i^{(-)R}$ using their relations

$$\begin{aligned}
\mathbf{a}_0^{(-)R} &= \mathbf{h}_0^{(-)R}, \\
\mathbf{a}_{\parallel}^{(-)R} &= \frac{1}{\sqrt{2}}(\mathbf{h}_+^{(-)R} + \mathbf{h}_-^{(-)R}), \\
\mathbf{a}_{\perp}^{(-)R} &= \frac{1}{\sqrt{2}}(\mathbf{h}_+^{(-)R} - \mathbf{h}_-^{(-)R}).
\end{aligned} \tag{19}$$

Here $\mathbf{a}_0^{(-)R}$ corresponds to longitudinal polarization of the J/ψ meson, and the other two coefficients correspond to polarizations of the J/ψ meson and h^+h^- system transverse to the decay axis: $\mathbf{a}_{\parallel}^{(-)R}$ for parallel polarization of the J/ψ and h^+h^- and $\mathbf{a}_{\perp}^{(-)R}$ for their perpendicular polarization.

In the SM, if we assume that only one diagram contributes to the decay and there is no direct CP violation, then the CKM weak phase only appears as $\frac{q}{p} = e^{-i\phi_s}$ for the B_s^0 decays and $\frac{q}{p} = e^{-2i\beta}$ for the B^0 decays. The $\mathbf{a}_i^{(-)}$ amplitudes only contain strong phases, so $\bar{\mathbf{a}}_i^R = \eta_i^R \mathbf{a}_i^R$, where η_i^R is CP eigenvalue of the i th transversity component for the intermediate state R . (Here $i = 0, \parallel, \perp$.) Note that for the h^+h^- system both C and P are given by $(-1)^{L_R}$, so the CP of the h^+h^- system is always even. The total CP of the final state is $(-1)^{L_B}$, since the CP of the J/ψ is also even. The final state CP parities for S, P, and D-waves are shown in Table 2.

Direct CP violation can also be considered, i.e. $\bar{\mathbf{a}}_i^R \neq \eta_i^R \mathbf{a}_i^R$. The complex coefficients can be parameterized as

$$\mathbf{a}_i^R = c_i^R(1 + b_i^R)e^{i(\delta_i^R + \phi_i^R)}, \quad \bar{\mathbf{a}}_i^R = \eta_i^R c_i^R(1 - b_i^R)e^{i(\delta_i^R - \phi_i^R)}, \tag{20}$$

where c_i^R , b_i^R , δ_i^R and ϕ_i^R are real numbers that can be determined in the experiment. Note that b_i^R and ϕ_i^R are CP violating, while c_i^R and δ_i^R are CP conserving. The direct CP asymmetry for a particular intermediate state R with the transversity i component is

$$A_{CP}(R)_i \equiv \frac{|\bar{\mathbf{a}}_i^R|^2 - |\mathbf{a}_i^R|^2}{|\bar{\mathbf{a}}_i^R|^2 + |\mathbf{a}_i^R|^2} = \frac{-2b_i^R}{1 + (b_i^R)^2}. \quad (21)$$

In the case that direct CP violation is present, the experiment measures an “effective” phase that is the sum of the CP violation due to the interference between mixing the decay and direct CP violation, given by

$$\phi_s^{\text{eff}}(R)_i = \phi_s + 2\phi_i^R, \text{ or } 2\beta^{\text{eff}}(R)_i = 2\beta + 2\phi_i^R. \quad (22)$$

To implement this procedure data need to be fit with the probability density functions (PDFs) given in Eqs. (16) and (17). The normalization can be computed by first integrating over t , θ_ℓ and χ analytically, then by using numerical integration for the remaining variables; the terms containing variable χ in Eqs. (12), (13) and (15) are zero when integrating over $\chi \in [-\pi, \pi]$. The data can be either flavour tagged or not [12]. In the latter case the two PDFs are averaged.

Without considering m_{hh} dependence, time-dependent angular analysis for ϕ_s determination in $B_s^0 \rightarrow J/\psi \phi$ decay [2–4] cannot distinguish between two ambiguous solutions, one that is $(\phi_s, \Delta\Gamma)$ and the other being $(\pi - \phi_s, -\Delta\Gamma)$, because the time-dependent differential decay rates are invariant under this transformation together with a similar transformation for the strong phases. This ambiguity has been resolved by the LHCb collaboration [13] using the P-wave ϕ interference with the K^+K^- S-wave [14] as a function of dikaon invariant mass as suggested in [15]. Our Dalitz-plot formalism automatically takes the strong phases as a function of m_{hh} into account in the complex function $A_R(m_{hh})$ in Eq. (18), and thus provides only one solution for (ϕ_s, Γ_s) without any ambiguity.

5 Conclusions

We have presented a method that can be used to extract the CP violating phase for neutral B meson decays into a spin-1 resonance that decays to a dilepton pair and a $\pi^+\pi^-$ or K^+K^- pair, using the full set of mass and angular variables. Thus CP violation can be measured using a much larger set of final states. For example, the K^+K^- mass range in $\bar{B}_s^0 \rightarrow J/\psi K^+K^-$ can be used including higher mass states such as the $f_2'(1525)$ [16].

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Appendix: Application to S- and P-waves in $B_s^0 \rightarrow J/\psi K^+ K^-$

Time-dependent angular analysis [3, 4] has been applied to $B_s^0 \rightarrow J/\psi \phi$ considering both a P-wave resonance, $\phi \rightarrow K^+ K^-$, and S-wave contamination [14]. Here we show that our formulation reduces to previously used expressions [15, 17] by considering only the ϕ mass region in $\bar{B}_s^0 \rightarrow J/\psi K^+ K^-$ decays. With this simplification, it is necessary to consider only S- and P-waves in the $K^+ K^-$ system. Eq. (8) can be rewritten as

$$\begin{aligned}\mathcal{H}_0 &= \frac{H_S}{\sqrt{3}} d_{0,0}^0(\theta_h) + H_0 d_{0,0}^1(\theta_h) = \frac{H_S}{\sqrt{3}} + H_0 \cos \theta_h, \\ \mathcal{H}_+ &= H_+ d_{-1,0}^1(\theta_h) = H_+ \frac{\sin \theta_h}{\sqrt{2}}, \\ \mathcal{H}_- &= H_- d_{1,0}^1(\theta_h) = -H_- \frac{\sin \theta_h}{\sqrt{2}},\end{aligned}\tag{23}$$

where H_S is the helicity amplitude for S-wave, and $H_{0,\pm}$ are the helicity amplitudes for P-wave ($\lambda = 0, \pm 1$). Then Eq. (12) can be expressed as

$$|A_f|^2 = \sum_{k=1}^{10} p_k G_k(\mathbf{\Omega}_{\text{hel}})\tag{24}$$

in terms of the amplitudes H , where $\mathbf{\Omega}_{\text{hel}}$ is short hand for the three angular variables $(\theta_h, \theta_\ell, \chi)$. The individual terms for p_k and $G_k(\mathbf{\Omega}_{\text{hel}})$ for $k=1-10$ are listed in Table 3.

Table 3: Definition of the functions p_k and $G_k(\mathbf{\Omega}_{\text{hel}})$ of Eq. (24).

k	p_k	$G_k(\mathbf{\Omega}_{\text{hel}})$
1	$ \frac{H_S}{\sqrt{3}} ^2$	$\sin^2 \theta_\ell$
2	$ H_0 ^2$	$\sin^2 \theta_\ell \cos^2 \theta_h$
3	$ H_+ ^2 + H_- ^2$	$\frac{1}{4}(1 + \cos^2 \theta_\ell) \sin^2 \theta_h$
4	$\mathcal{R}e(\frac{H_S}{\sqrt{3}} H_0^*)$	$2 \sin^2 \theta_\ell \cos \theta_h$
5	$\mathcal{R}e(H_+ H_-^*)$	$-\frac{1}{2} \sin^2 \theta_\ell \sin^2 \theta_h \cos 2\chi$
6	$\mathcal{I}m(H_+ H_-^*)$	$\frac{1}{2} \sin^2 \theta_\ell \sin^2 \theta_h \sin 2\chi$
7	$\mathcal{R}e[\frac{H_S}{\sqrt{3}} (H_+^* + H_-^*)]$	$\frac{1}{2} \sin 2\theta_\ell \sin \theta_h \cos \chi$
8	$\mathcal{I}m(\frac{H_S}{\sqrt{3}} (H_+^* - H_-^*))$	$\frac{1}{2} \sin 2\theta_\ell \sin \theta_h \sin \chi$
9	$\mathcal{R}e(H_0 (H_+^* + H_-^*))$	$\frac{1}{4} \sin 2\theta_\ell \sin 2\theta_h \cos \chi$
10	$\mathcal{I}m(H_0 (H_+^* - H_-^*))$	$-\frac{1}{4} \sin 2\theta_\ell \sin 2\theta_h \sin \chi$

The functions can be expressed using transversity amplitudes by using the relations between helicity and transversity amplitudes (A) [18], and the relations between helicity

$(\theta_h, \theta_\ell, \chi)$, $\mathbf{\Omega}_{\text{hel}}$, and transversity angles $(\psi_{\text{tr}}, \theta_{\text{tr}}, \phi_{\text{tr}})$, $\mathbf{\Omega}_{\text{tr}}$. The amplitudes relations are

$$\begin{aligned} A_S &= H_S, \\ A_0 &= H_0, \\ A_{\parallel} &= \frac{1}{\sqrt{2}}(H_+ + H_-), \\ A_{\perp} &= \frac{1}{\sqrt{2}}(H_+ - H_-), \end{aligned} \quad (25)$$

and the angular relationships are

$$\begin{aligned} \cos \psi_{\text{tr}} &= \cos \theta_h, \\ \sin \theta_{\text{tr}} \cos \phi_{\text{tr}} &= -\cos \theta_\ell, \\ \sin \theta_{\text{tr}} \sin \phi_{\text{tr}} &= -\sin \theta_\ell \cos \chi, \\ \cos \theta_{\text{tr}} &= \sin \theta_\ell \sin \chi. \end{aligned} \quad (26)$$

The amplitudes \bar{A}_i are related to A_i as

$$\frac{q}{p} \frac{\bar{A}_i}{A_i} = \eta_i e^{-i\phi_s}, \quad (27)$$

where η_i is CP eigenvalue of the i component; η_S and $\eta_{\perp} = -1$, and η_0 and $\eta_{\parallel} = 1$. We express the amplitudes as functions of either the helicity distributions or transversity distributions as the sums

$$|A_f^{(-)}|^2 = \sum_{k=1}^{10} q_k^{(-)} g_k(\mathbf{\Omega}_{\text{hel}}) = \sum_{k=1}^{10} q_k^{(-)} g_k(\mathbf{\Omega}_{\text{tr}}). \quad (28)$$

From Eqs. (15) and (27), we compute the interference terms as

$$\frac{q}{p} A_f^* \bar{A}_f = e^{-i\phi_s} \left(\sum_{k=1}^{10} r_k g_k(\mathbf{\Omega}_{\text{tr}}) \right). \quad (29)$$

Each term is listed in Table 4. In Ref. [3] the time-dependent and angular-dependent rate for $B_s^0 \rightarrow J/\psi \phi$ is written as

$$\frac{d^4\Gamma}{dt d\mathbf{\Omega}_{\text{tr}}} \propto \sum_{k=1}^{10} h_k(t) f_k(\mathbf{\Omega}_{\text{tr}}), \quad (30)$$

where the time-dependent function

$$h_k(t) = N_k e^{-\Gamma t} \left[a_k \cosh \frac{\Delta\Gamma t}{2} + c_k \cos(\Delta m t) + b_k \sinh \frac{\Delta\Gamma t}{2} + d_k \sin(\Delta m t) \right], \quad (31)$$

and the $f_k(\mathbf{\Omega}_{\text{tr}})$ represent angular-dependent functions. Comparing with Eq. (5) it can be seen that a_k corresponds to $\frac{|A_f|^2 + |\bar{A}_f|^2}{2}$, c_k to $\frac{|A_f|^2 - |\bar{A}_f|^2}{2}$, b_k to $-\mathcal{R}e(\frac{q}{p} A_f^* \bar{A}_f)$ and d_k to $-\mathcal{I}m(\frac{q}{p} A_f^* \bar{A}_f)$. Using Table 4, we find the same equations as shown in Ref. [3].

Table 4: Definition of the functions used in Eqs. (28) and (29). When two signs appear, the upper one corresponds to q_k and the lower to \bar{q}_k .

\bar{q}_k	r_k	$g_k(\mathbf{\Omega}_{\text{hel}})$	$g_k(\mathbf{\Omega}_{\text{tr}})$
$ A_0 ^2$	$ A_0 ^2$	$\sin^2 \theta_\ell \cos^2 \theta_h$	$\cos \psi_{\text{tr}} (1 - \sin^2 \theta_{\text{tr}} \cos^2 \phi_{\text{tr}})$
$ A_{\parallel} ^2$	$ A_{\parallel} ^2$	$\frac{1}{2}(1 - \sin^2 \theta_\ell \cos^2 \chi) \sin^2 \theta_h$	$\frac{1}{2} \sin^2 \psi_{\text{tr}} (1 - \sin^2 \theta_{\text{tr}} \sin^2 \phi_{\text{tr}})$
$ A_{\perp} ^2$	$- A_{\perp} ^2$	$\frac{1}{2}(1 - \sin^2 \theta_\ell \sin^2 \chi) \sin^2 \theta_h$	$\frac{1}{2} \sin^2 \psi_{\text{tr}} \sin^2 \theta_{\text{tr}}$
$ \frac{A_S}{\sqrt{3}} ^2$	$- \frac{A_S}{\sqrt{3}} ^2$	$\sin^2 \theta_\ell$	$1 - \sin^2 \theta_{\text{tr}} \cos^2 \phi_{\text{tr}}$
$\mathcal{R}e(A_0^* A_{\parallel})$	$\mathcal{R}e(A_0^* A_{\parallel})$	$\frac{1}{2\sqrt{2}} \sin 2\theta_\ell \sin 2\theta_h \cos \chi$	$\frac{1}{2\sqrt{2}} \sin 2\psi_{\text{tr}} \sin^2 \theta_{\text{tr}} \sin 2\phi_{\text{tr}}$
$\pm \mathcal{I}m(A_0^* A_{\perp})$	$i \mathcal{R}e(A_0^* A_{\perp})$	$-\frac{1}{2\sqrt{2}} \sin 2\theta_\ell \sin 2\theta_h \sin \chi$	$\frac{1}{2\sqrt{2}} \sin 2\psi_{\text{tr}} \sin 2\theta_{\text{tr}} \cos \phi_{\text{tr}}$
$\pm \mathcal{R}e(A_0^* \frac{A_S}{\sqrt{3}})$	$-i \mathcal{I}m(A_0^* \frac{A_S}{\sqrt{3}})$	$2 \sin^2 \theta_\ell \cos \theta_h$	$2 \cos \psi_{\text{tr}} (1 - \sin^2 \theta_{\text{tr}} \cos^2 \phi_{\text{tr}})$
$\pm \mathcal{I}m(A_{\parallel}^* A_{\perp})$	$i \mathcal{R}e(A_{\parallel}^* A_{\perp})$	$\frac{1}{2} \sin^2 \theta_\ell \sin^2 \theta_h \sin 2\chi$	$-\frac{1}{2} \sin \psi_{\text{tr}} \sin 2\theta_{\text{tr}} \sin \phi_{\text{tr}}$
$\pm \mathcal{R}e(A_{\parallel}^* \frac{A_S}{\sqrt{3}})$	$-i \mathcal{I}m(A_{\parallel}^* \frac{A_S}{\sqrt{3}})$	$\frac{1}{\sqrt{2}} \sin 2\theta_\ell \sin \theta_h \cos \chi$	$\frac{1}{\sqrt{2}} \sin \psi_{\text{tr}} \sin^2 \theta_{\text{tr}} \sin 2\phi_{\text{tr}}$
$\mathcal{I}m(A_{\perp}^* \frac{A_S}{\sqrt{3}})$	$-\mathcal{I}m(A_{\perp}^* \frac{A_S}{\sqrt{3}})$	$\frac{1}{\sqrt{2}} \sin 2\theta_\ell \sin \theta_h \sin \chi$	$-\frac{1}{\sqrt{2}} \sin \psi_{\text{tr}} \sin 2\theta_{\text{tr}} \cos \phi_{\text{tr}}$

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